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THERMAL INSTABILITY OF THE HYDROGEN-BURNING SHELL IN NONDEGENERATE STARS

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ABSTRACT

An investigation is made of thermal instability in the hydrogen-burning shell of stars of moderate to high mass evolving from the end of core hydrogen burning to the early stages of core helium burning, with the help of an approximate analytic criterion for thermal instability and full nonlinear numerical calculations of stellar evolution. Two different assumptions about the chemical evolution in the stars of highest mass are adopted. Thermal pulses are found to develop only in very massive stars having convective intermediate zones, when the hydrogen profile in the shell is sufficiently steep and the shell is still moderately thick.

I. INTRODUCTION

Following the discovery of thermal (secular) instability in the helium-burning shell of stars (Schwarzschild and Härm 1965; Weigert 1965, 1966), the possibility of thermally unstable hydrogen-burning shells has been mentioned from time to time. Since an example of one has never been found except in the special case of a thin hydrogen envelope on top of a degenerate white dwarf (e.g., Giannone and Weigert 1967; Kippenhahn, Thomas, and Weigert 1968), most authors have concluded that hydrogen-burning shells is normally evolving stars are probably thermally stable. The fact that high radiation pressure tends to oppose thermal instability in a burning shell has also seemed to preclude its occurrence in massive stars (Hōshi 1968; Dennis 1971).

In § II of the present paper, it will be shown on analytical grounds that thermal instability is a distinct possibility in the hydrogen-burning shell of stars before the exhaustion of core helium, even in stars of high mass. Detailed numerical calculations of stellar evolution in § III will indicate the approximate stellar masses and conditions under which thermal instability is actually found to occur.

II. ANALYTIC CRITERION FOR THERMAL INSTABILITY

Following the original analysis of Schwarzschild and Härm (1965), a number of approximate criteria for the thermal instability of a nuclear-burning shell have been worked out (Giannone and Weigert 1967; Kippenhahn 1967; Hōshi 1968; Unno 1970; Dennis 1971; Henyey and Ulrich 1972). Most of these criteria are too complicated to apply for our purpose here, but Hōshi's criteria seem to be appropriate. There are actually two criteria to consider. First, a positive entropy perturbation gives rise to a temperature increase in the burning shell if

$$\frac{\Delta r}{r} < \frac{\beta(32 - 24\beta - 3\beta^2)}{2(4 - 3\beta)^2|Q|}, \tag{1}$$

where Δr is the radial thickness of the shell, β is the ratio of gas pressure to total pressure, and Q is a factor of order unity. Inequality (1), as given by Hōshi (originally

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by Schwarzschild and Härm), has been modified to include radiation pressure; the inequality is fulfilled only in relatively thin shells. Second, the entropy gain in the shell due to enhanced nuclear energy generation outweighs the entropy loss due to increased flux divergence if

$$\frac{\Delta M}{M} > 6\left(\frac{2}{e}\right)^2 \frac{4ac(4\pi r^2)^2 T^4}{3\kappa L_s M \nu}, \tag{2}$$

where ΔM is the mass thickness of the shell, T is the temperature, κ is the (constant) shell opacity, L_s is the luminosity produced by the shell source, and ν is the temperature exponent of the rate of nuclear energy generation, ϵ. We shall call the right-hand side of this inequality $(\Delta M/M)_{\rm crit}$. Inequality (2), as given by Hōshi, requires no modification to include radiation pressure because β is approximately constant across the shell. To obtain an estimate of the mass ΔM contained in the shell, we shall arbitrarily define the top and the bottom of the shell as the points where ϵ has dropped to $\frac{1}{5}$ times its peak value.

An alternative formulation of the second criterion is the original one due to Schwarzschild and Härm, whose assumptions about the profiles of the various perturbations have been numerically verified both by themselves and by others (see Rose 1966, fig. 1). A slight generalization of their work to include both radiation pressure and a nonzero flux at the base of the shell yields identically their original criterion, which was given in terms of the *temperature* difference across the shell (their eq. [12]). Transformation of this criterion into one for the *mass* difference is easily effected by introducing, in difference form, the equation of radiative transfer (their eq. [1]). This results in an expression identical to criterion (2) except for the occurrence of a factor 8 in place of $6(2/e)^2$. This difference is negligible in view of the necessary approximations used in both derivations.

Several sequences of stellar models have been published in the literature with enough information given to evaluate criterion (2) at the hydrogen-burning shell. These sequences are for 5 $M_\odot$ (Polak 1962), 7 $M_\odot$ (Hofmeister, Kippenhahn, and Weigert 1964), and 30 $M_\odot$ (Stothers 1964; Kotok 1966). However, an *analytic* expression for the mass ΔM contained between the two layers in which ϵ has dropped off by a factor f from its peak value can be derived if the profile of ϵ across the shell is known. The models just quoted indicate that ϵ is approximately symmetrical about the shell peak, and therefore our previous (Stothers 1966) result for the dependence of ϵ on $M(r)$ can be used (by doubling it) to obtain

$$\frac{\Delta M}{M} = \left(\frac{6 \ln f}{\nu + 3}\right) \frac{4\pi r^3 \rho}{3M}. \tag{3}$$

Expression (3) is found to yield slightly less than the exact mass of the shell, derived from published models, if f is chosen to be 5. But considerable leeway in f is possible, and so expression (3) should be used with caution.

Application of the instability criteria (1) and (2), with the help of equation (3) and the published models, indicates that thermal instability becomes increasingly favored as the thick hydrogen shell narrows. Too early in the evolution and the thick shell is stable, for the same reason that a nuclear-burning core is stable (Ledoux 1958); specifically, the flux produced by the shell is too small for its mass. But too late in the evolution and the narrow shell is stable, because it loses its flux too rapidly. Hence our interest focuses on the stages of evolution immediately prior to core helium burning, when criteria (1) and (2) seem to be best satisfied.

The *e*-folding time of thermal instability is given variously by different authors.

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However, we shall consistently use Hōshi's expression, modified to include radiation pressure,

$$\tau_s = \frac{32 - 24\beta - 3\beta^2}{2\beta^2}\left(\frac{kT}{\mu H}\right)\Big/\nu\epsilon\,, \tag{4}$$

which is to be evaluated at the peak of nuclear energy production in the shell.

III. NONLINEAR NUMERICAL MODELS

In order to test for the actual occurrence of thermal instability in hydrogen-burning shells, evolutionary sequences of stellar models for 5, 7, 10, 15, 30, and 60 $M_\odot$ have been computed by using a numerical relaxation method (the ingredients of our stellar-evolution program will be described in a later paper). With an initial chemical composition of $X_e = 0.739$, $Z_e = 0.021$, and $X_{\rm CN} = \frac{1}{3}Z_e$, each sequence has been evolved from the end of core hydrogen burning to the earliest stages of core helium burning. The main phase of hydrogen burning has been covered in a previous paper (Stothers 1972).

Two different schemes of evolution have been assumed for the models of high mass. In the notation of Stothers (1970, 1972), adoption of the Ledoux criterion for convective instability gives rise to a stable radiative intermediate zone of smoothly varying hydrogen-helium abundance until late in the evolution at all masses (scheme R), while adoption of the Schwarzschild criterion gives rise, in masses heavier than $\sim 10\ M_\odot$, to a rapidly growing semiconvective zone, whose lower boundary becomes fully convective as time goes on and is separated from an underlying radiative zone by a sharp hydrogen-helium discontinuity (scheme N2). For the models based on scheme N2, we have simply adopted the hydrogen profile prevailing at the stage of core hydrogen exhaustion, as determined by Simpson (1971) in the case of 30 $M_\odot$ and by Iben (1966) in the case of 15 $M_\odot$. (Iben's treatment of the intermediate zones was not quite that of scheme N2 but very close to it.) The case of 60 $M_\odot$ is discussed below. In none of our models has further convective mixing been permitted to occur outside the convective core, but the hydrogen distribution in the shell has otherwise been allowed to change in accordance with the prevailing nuclear reaction rate. The hydrogen profiles of the two schemes are displayed in figure 1, and clearly show two very different hydrogen gradients in and above the hydrogen-burning shell when it has burned out to the indicated mass fraction. Thermal runaways have been looked for in our models by employing a series of very small time steps ($\ll \tau_s$) at various intervals during the evolution after core hydrogen burning.

The thick shell which is set up during central hydrogen exhaustion is found to be thermally stable for all masses, as expected according to criteria (1) and (2). At this stage the response of the shell to a small, arbitrary temperature perturbation is a rapidly damped, pulsed oscillation. As the shell source narrows in size, the ratio $\Delta M/(\Delta M)_{\rm crit}$ increases and attains a maximum value of 1.8–0.6 for masses of 5–60 $M_\odot$ respectively; this maximum occurs at a hotter central (and shell) temperature for higher masses but is independent of the adopted hydrogen profile. Subsequently (but usually before core helium ignition) $\Delta M/(\Delta M)_{\rm crit}$ begins to decrease. Despite the formal indication of possible thermal instability according to criterion (2), the shells of all the models based on scheme R are found to remain thermally stable. Schwarzschild and Härm (1965) and Henyey and Ulrich (1972) have tested models for 1 $M_\odot$ and 2 $M_\odot$, respectively, just prior to core helium ignition, and have found that the hydrogen shells of these models are also thermally stable. However, if scheme N2 is adopted, thermal instability does, in some cases, develop [causing $\Delta M/(\Delta M)_{\rm crit}$ to increase during the thermal runaway]. Such an instability may have developed in the 50 $M_\odot$ sequence of Morris (1970), which was based on scheme N2 and exhibited a

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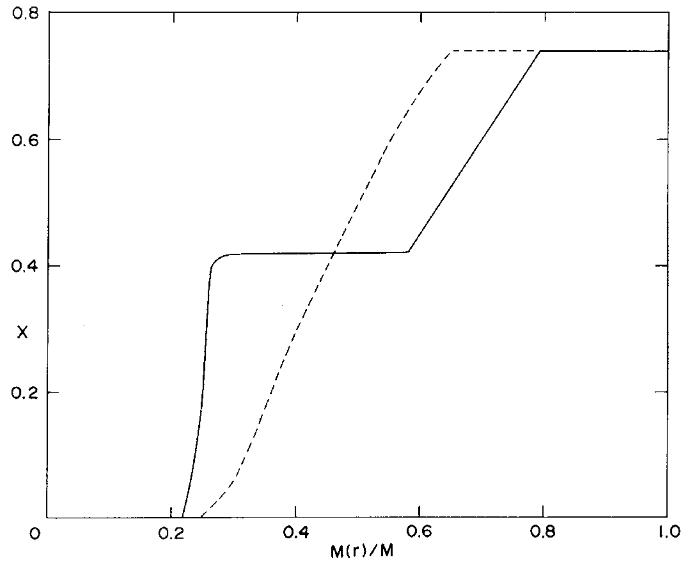

FIG. 1.—Hydrogen profile for scheme R (*dashed line*) and scheme N2 (*solid line*) in a star of 30 $M_{\odot}$ at the stage of maximum tendency to thermal instability in the hydrogen-burning shell. Note the difference in hydrogen gradient across the shell region (region of small X).

mild, damped oscillation of the surface luminosity. We can therefore say that criterion (2) provides, to within a factor of 2, a necessary, but not sufficient, condition for thermal instability, in the range of models tested here.

Discovery of thermal instability in some of the more massive models is rather surprising because the temperature exponent of the rate of nuclear energy generation at the shell peak is smaller for higher masses, the flux produced by the shell source escapes from a larger surface area, and the high radiation pressure tends to make the rise time slower and thermal readjustments more nearly homologous. Nevertheless, the steeper hydrogen gradient across the burning shell in scheme N2 causes a larger flux to be produced locally than in scheme R, hence inducing a thermal instability of the following general character: (1) small-amplitude thermal pulses develop in the hydrogen shell, each pulse separated by a long time interval whose duration is somewhat greater for larger amplitudes; (2) the pulse width is rather broad because the maximum amplitude attained is never very great; and (3) the pulses persist, with decaying amplitude, into the earliest stages of core helium depletion, whereupon they gradually disappear as the shell source narrows. Qualitatively speaking, the time profile of a pulse cycle in the hydrogen-burning shell of a massive star (fig. 2) resembles that of a pulse cycle in the helium-burning shell of a low-mass star (e.g., Rose 1966, fig. 4). However, the damping mechanisms present at high mass mentioned above severely limit the growth of the pulse amplitudes, whose reality we have been able to confirm in each model sequence by rerunning the sequence with different starting conditions, time steps, and criteria of computational accuracy. That the pulses do not affect the overall evolution has been verified by again rerunning each sequence with sufficiently long time steps that the pulses are avoided.

Detailed results for the most unstable models of 15, 30, and 60 $M_{\odot}$ are shown in table 1, where a subscript s refers to the shell peak, "delay time" means the time lag between the maximum of shell temperature (T_s) and the minimum of surface luminosity (L), "cycle time" means the time interval between pulse peaks, and $q = M(r)/M$. The models for 60 $M_{\odot}$ based on scheme N2 were calculated by arbitrarily imposing the hydrogen profile (scaled in mass fraction) from the models at 30 $M_{\odot}$. Two further

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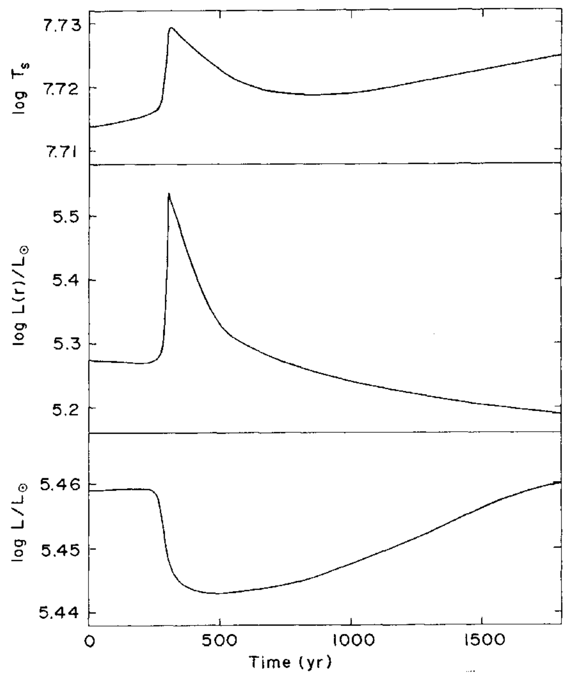

FIG. 2.—A pulse cycle for the thermally unstable hydrogen-burning shell in a star of 30 $M_\odot$ evolving according to scheme N2. The panels show, respectively, logarithms of the shell peak temperature, shell peak luminosity, and surface luminosity. The maxima and steepest segments of the curves are somewhat uncertain. Note that the overall evolutionary changes show up even on the short timescale of a single pulse cycle since the star is evolving on a gravitational (Kelvin) timescale.

sequences for 60 $M_\odot$ were calculated with a steeper hydrogen gradient at the shell and with a smaller shell-peak mass fraction, respectively. In the unstable sequence for 15 $M_\odot$, only four extremely feeble pulses occurred altogether.

The amplitude and even the occurrence of the thermal pulses are found to be very sensitive to the hydrogen gradient and location of the burning shell. Although a steep hydrogen gradient across the shell promotes thermal instability, *too* steep a gradient (i.e., too thin a shell) tends to quench the instability. However, even in the latter case, if the shell is placed farther inward in mass fraction, it becomes unstable again because its radiating surface area is smaller. It seems likely that these remarks about the hydrogen shell apply equally well to a helium shell, and may account for the fact that different authors sometimes obtain different results for the thermal instability of a helium shell in ostensibly the same stellar model. However, a helium shell when first formed is already located at (or very near) a helium discontinuity, whereas the hydrogen shell has to consume its way outward to the hydrogen discontinuity, and at such a rate that the shell achieves the proper thickness on arrival if it is to be thermally unstable. Thus it is essential, in searching for realistic thermal runaways, to calculate very carefully the distribution of burning fuel throughout the shell (and especially at its base). Machine interpolation of mass zones and convective modifications during a pulse (which we have ignored) must also be treated with great care, as must the distribution of fuel farther out in the star which the burning shell will reach in time. However, the exaction of greater precision in our models at the present time seems unwarranted because of existing uncertainties about the convective and semiconvective mixing, initial chemical composition, and (possibly) rotational effects.

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TABLE 1

CHARACTERISTICS OF THE EVOLUTIONARY SEQUENCES OF MASSIVE STARS AT THE STAGE OF MAXIMUM TENDENCY TO THERMAL INSTABILITY IN THE HYDROGEN-BURNING SHELL

$M/M_\odot$	15	15	30	30	60	60	60	60
Scheme	R	"N2"	R	N2	R	N2	N2	N2
q_s	0.18	0.20	0.26	0.23	0.38	0.38	0.38	0.35
$(\Delta q)_{\rm shell}$	0.04	0.03	0.05	0.02	0.05	0.04	0.03	0.02
$\langle dX/dq \rangle_{\rm shell}$	5	10	3	10	2	10	20	20
$(1-\beta)_s$	0.25	0.27	0.50	0.52	0.70	0.66	0.67	0.67
Largest $\delta \log T_s$	...	0.001	...	0.016	...	0.010	...	0.003
Largest $\delta \log (L/L_\odot)$	...	-0.000	...	-0.014	...	-0.005	...	-0.001
τ_s (10^2 yr)	...	3	...	3	...	12	...	5
Delay time (10^2 yr)	...	<1	...	~ 1.5	...	~ 0.7	...	1
Cycle time (10^3 yr)	...	0.5	...	~ 2	...	~ 1.5	...	0.3

In summary, we have shown that, under certain conditions, massive stars with convective intermediate zones can experience thermal pulses of small amplitude in a moderately thick hydrogen-burning shell. The sensitivity of the pulses to the hydrogen gradient and location of the burning shell may be related to the similar sensitivity of the evolutionary loops in the H-R diagram after core hydrogen exhaustion (e.g., Stothers and Chin 1968). Observational confirmation or rejection of the pulses could, in principle, be used to determine which scheme of evolution is correct, but the practical difficulty of observing changes of (at most) hundredths of a magnitude over a period of decades seems to obviate this course. In a subsequent paper, we shall consider the possible thermal instability of helium-burning shells in massive stars.

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